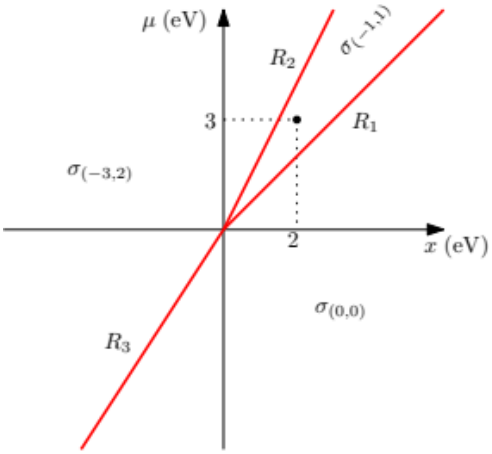
Dr. Mounir Nisse and Dr. Lim Yen Kheng Discover Applications of Tropical Algebraic Geometry in Statistical Mechanics

Associate Professor Dr. Mounir Nisse and Associate Professor Dr. Lim Yen Kheng from the School of Mathematics and Physics recently published a journal article titled "The zero-temperature limit of grand canonical ensembles via tropical geometry" in Analysis and Mathematical Physics (SCIE-Tier 1, 68/330 in Mathematics, 2020 impact factor: 1.548).

In this paper, a new application of tropical geometry was found in statistical mechanics. The field of tropical geometry is a study of numbers in the tropical semi-ring, which is a set of numbers with 'addition' and 'multiplication' defined in a special way. These special operations can be obtained from a limiting procedure of standard operations on numbers.

The key idea for this paper is the realisation that this limiting procedure is mathematically almost identical to something in statistical mechanics; namely taking the zero-temperature limit of a system! Therefore, the rich results of tropical algebraic geometry can now be carried over to study physical systems at ultracold temperatures.

A demonstrative example is the case of molecular surface adsorption, where on the surface of a material contains 'sites' which can hold up to two particles each. Therefore there are three possible states of zero, one, or two particles on each site. The actual states that occur depend on the occupation energy (x), chemical potential (μ), and temperature of the environment. The figure below shows the zero temperature limit of the possible distribution of states, and we see the definitive shape of a tropical curve.



These results open up a new window of possibilities where sophisticated results from the field of tropical geometry may be carried over to solve problems in low-temperature physics. The paper can be found at https://doi.org/10.1007/s13324-021-00555-8.

Associate Professor Dr. Mounir Nisse was an invited researcher at IHES, Bures-sur-Yvette, France before joining Xiamen University Malaysia. His primary focus of research lies in the areas of complex algebraic geometry, computational algebraic geometry, tropical geometry, complex analysis, combinatorics, deformation of singularities, (co)amoebas of complex varieties, and mirror symmetry.

Associate Professor Dr. Lim Yen Kheng received his PhD from National University of Singapore in December 2015, where he then worked as an instructor until August 2019 before joining Xiamen University Malaysia. He has published more than 20 papers in journals including Physical Review D, Classical and Quantum Gravity, *and the* Journal of High Energy Physics. *His research interests are black holes, general relativity, and theoretical/mathematical physics.*