

2026

厦大杯中学物理比赛

XMUM Cup Physics Competition for Secondary School

Question	Acceptable answers		Question	Acceptable answers
1	13		16	0.53
2	92		17	3.8
3	1.5		18	8.2
4	7.2		19	0.36
5	4.5		20	8.5
6	8.8		21	0.62
7	0.79		22	2.3
8	0.16		23	5.4
9	2.7		24	40
10	0.43		25	2.0
11	2.7		26	0.86
12	4.8		27	6.6
13	0.83		28	1.9
14	98		29	60
15	10		30	0.48

Topics	Questions	No. Questions
Mechanics	1-10	10
Thermal Physics	11-14	4
Electromagnetism	15-21	7
Electronics Circuit	22-23	2
Modern Physics	27-30	4
Optics	24-26	3
	Total	30

Question 1

A helium balloon rises vertically with a constant speed of 5.0 m s^{-1} . At the same time, a car moves horizontally to the east with a constant speed of 12 m s^{-1} . Determine the speed (in m s^{-1}) of the balloon relative to the car.

Ans: 13

$$\text{Relative velocity } v_{rel} : \sqrt{12^2 + 5^2} = 13 \text{ m s}^{-1}$$

Question 2

A 4.0 kg crate is pushed slowly up an inclined rough surface at 60° to the horizon. The kinetic coefficient of friction between the crate and the surface is 0.30. Determine the minimum work (in joules) required to push the crate up a vertical height of 2.0 m.

Ans: 92

Minimum work done is

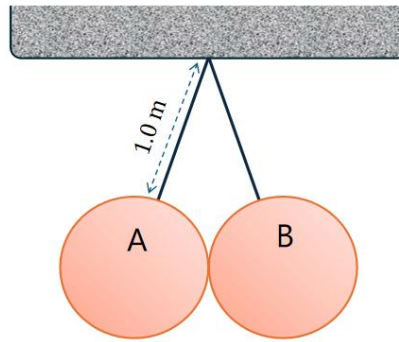
$$W = mgh + fs$$

where $f = \mu N$ is the friction, $N = mg \cos 60^\circ$ being the normal force of the crate on the slope. Furthermore, $s = \frac{h}{\sin 60^\circ}$ is the distance of the crate traveling along the slope. This yields

$$W = mgh + \mu mg \cos 60^\circ \left(\frac{h}{\sin 60^\circ} \right) = 92 \text{ J}$$

Question 3

Two identical hard spheres, A and B are suspended by strings of length 1.0 m as shown in the figure. The mass and the diameter of a sphere are 0.9 kg and 0.4 m, respectively. Determine the magnitude of the contact force (in newtons) exerted by sphere B on sphere A.

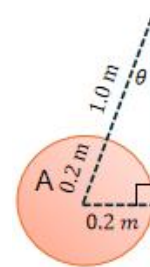


Ans: 1.5

For sphere A

$$T \cos \theta = mg \text{ and } T \sin \theta = N$$

where the θ is given by the following graph:



Therefore, the force of B act on A is

$$N = mg \tan \theta = 0.9 \times 9.8 \times \frac{0.2}{\sqrt{1.2^2 - 0.2^2}} = 1.49 \text{ N}$$

Question 4

In a race, Car A reached its maximum speed of 50 m s^{-1} when it was 300 m away from the finish line. At that moment, Car B is 100 m behind Car A and moving at a speed of 45 m s^{-1} . Determine the minimum constant acceleration (in ms^{-2}) required for Car B to reach the finish line no later than Car A.

Ans: 7.2

For Car A, we have

$$x_A(t) = x_A(0) + v_A t$$

with $x_A(0) = -300 \text{ m}$ and $v_A = 50$, where we have set the finish line at $x = 0$. Car A will reach the finish line at

$$x_A(t) = 0 \rightarrow t = -\frac{x_A(0)}{v_A} = \frac{300}{50} \text{ s} = 6 \text{ s}$$

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Car B started to accelerate at $x_B(0) = -400$ with velocity $v_B = 45$ m/s. this gives

$$x_B(t) = x_B(0) + v_B t + \frac{1}{2} a t^2$$

By requiring car B reach the finish line at $t = 6$ s, we arrive at

$$x_B(t = 6\text{ s}) = 0$$

Therefore, the minimum a is

$$-400 + 45 * 6 + \frac{1}{2} a 6^2 = 0 \rightarrow a = \frac{400 - 270}{0.5 * 6^2} = 7.22 \text{ ms}^{-2}$$

Question 5

Two particles move along the x -axis. Particle A has a mass 2.0 kg and is initially located at $x = -3.0$ m, moving to the right with a speed 5.0 m s^{-1} . Particle B has a mass 4.0 kg and is initially located at $x = 6.0$ m, moving to the left with a speed 2.0 m s^{-1} . Find the position of the centre of mass after 4.5 s.

Ans: 4.5

Initial Centre of mass

$$x_{cm} = \frac{2(-3) + 4(6)}{6} = 3 \text{ m}$$

Velocity

$$v_{cm} = \frac{2(5) + 4(-2)}{6} = \frac{1}{3} \text{ m/s}$$

After 4.5 s

$$x_{cm} = 3 + \frac{1}{3}(4.5) = \boxed{4.5 \text{ m}}$$

Question 6

A small block of mass 0.3 kg moves in circular motion on a smooth surface with a 30° inclination. It is attached to a light string of length R . The block moves with the minimum speed required to keep the string taut at the highest point of the motion. Determine the tension (in newtons) in the string when the block reaches its lowest point.

Ans: 8.8

At the highest point, the minimum speed occurs when the tension in the string is zero. Therefore,

$$\frac{mv_t^2}{R} = mg \sin 30^\circ \rightarrow v_t^2 = Rg \sin 30^\circ$$

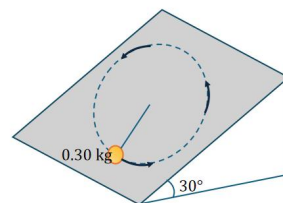
Conservation of energy:

$$\frac{1}{2}mv_b^2 = \frac{1}{2}mgR \sin 30^\circ + 2mgR \sin 30^\circ$$

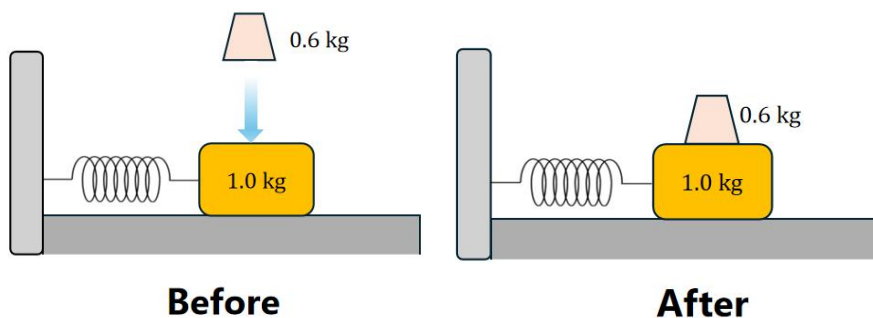
$$v_b^2 = \frac{5}{2}gR$$

$$T_b - mg \sin 30^\circ = \frac{mv_b^2}{R}$$

$$T = \frac{6}{2}mg = 8.82 \text{ N}$$

**Question 7**

A block of mass 1.0 kg is in simple harmonic oscillation with a maximum displacement D_0 . A second block of mass 0.6 kg falls vertically and sticks to the oscillating block as it passes through the equilibrium position. If the new maximum displacement of the blocks becomes D , what is the ratio of maximum displacement D/D_0 ?



Ans: 0.79

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Before collision, the oscillating block has a horizontal velocity v . The falling block horizontal velocity is 0. After collision, the two blocks share the same horizontal velocity V .

By conservation of momentum

$$Mv = (m+M)V$$

Hence

$$V/v = \frac{M}{m+M}$$

Maximum displacement before collision

$$v = \omega_0 D_0 = D_0 \sqrt{\frac{k}{M}}$$

Similarly, maximum displacement after collision

$$V = \omega D = D \sqrt{\frac{k}{M+m}}$$

Amplitude ratio

$$\frac{D}{D_0} = \frac{V/\omega}{v/\omega_0} = \sqrt{\frac{M}{m+M}} = \sqrt{\frac{1}{1.6}} = 0.79$$

Question 8

A celestial body of mass $m_0 = 10^{23}$ kg explodes and shatters into two fragments of masses m_1 and m_2 with $m_2/m_1 = 0.01$. The two fragments come to rest relatively after travelling a total separation of $R = 500\,000$ km due to gravitational forces. If the size of the fragments is negligible compared to R , determine the magnitude of the initial velocity (in km s^{-1}) of m_2 relative to the center of mass. (Newton gravitational constant: $G = 6.7 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)

Ans: 0.16

Conservation of momentum gives

$$m_2 v_2 + m_1 v_1 = 0 \rightarrow 0.01 v_2 + v_1 = 0 \rightarrow v_2 = -100 v_1$$

The relative velocity between the two fragments is

$$v_r = v_2 - v_1 = 1.01 v_2$$

Confidential

Conservation of energy gives

$$\frac{1}{2}\mu v_r^2 = \frac{G m_1 m_2}{R}$$

$$\frac{1}{2}\left(\frac{m_1 m_2}{m_0}\right) v_r^2 = \frac{G m_1 m_2}{R}$$

This yields

$$v_r^2 = \frac{2Gm_0}{R}$$

$$1.01v_2 = \sqrt{\frac{2(6.7 \times 10^{-11})10^{23}}{5 \times 10^8}}$$

The initial speed of m_2 is

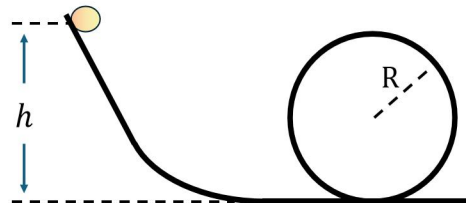
$$v_2 = 162 \text{ m s}^{-1}$$

Question 9

A solid sphere with mass m and radius r is released from rest and *rolls without slipping* down a track into a vertical loop of radius R . The moment of inertia of a solid sphere is $I = \frac{2}{5}mr^2$. If the minimum release height that required for the sphere to complete the loop is h . Then, what is h/R ?

Ans: 2.7

The velocity at the top of the loop needs



$$v_{top}^2 = gR$$

Energy conservation gives

$$mgh = mg(2R) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \text{with } I = \frac{2}{5}mr^2$$

Using $v = r\omega$

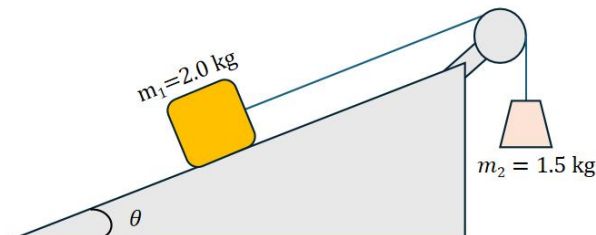
$$h = 2R + \frac{1}{2}R + \frac{1}{2} \frac{2}{5}R$$

Hence,

$$\frac{h}{R} = \frac{27}{10} = 2.7$$

Question 10

A block of mass $m_1 = 2.0$ kg lies on a $\theta = 30^\circ$ incline and is connected by a light string over a frictionless pulley to a hanging mass $m_2 = 1.5$ kg. The coefficients of static and kinetic friction are $\mu_s = 0.25$, $\mu_k = 0.20$, respectively. Determine the magnitude of the acceleration of the mass m_2 . (in ms^{-2})



Ans: 0.43

Assuming that, m_2 moves DOWNWARD, we have

$$\begin{aligned} m_2 g - T &= m_2 a \\ T - m_1 g \sin \theta - \mu_k m_1 g \cos \theta &= m_1 a \end{aligned}$$

where the T (tension) and a (acceleration) are positive.

This gives

$$\begin{aligned} T &= \frac{m_1 m_2 g (1 + \sin \theta + \mu \cos \theta)}{m_1 + m_2} \\ a &= \frac{m_2 - m_1 (\sin \theta + \mu \cos \theta)}{m_1 + m_2} g \end{aligned}$$

If $\mu = \mu_s$, $a > 0$, the weight of m_2 is large enough to overcome the static friction.

Now let $\mu = \mu_k$

$$a = \frac{1.5 - 2(\sin 30 + 0.2 \cos 30)}{2 + 1.5} 9.8 = 0.43 \text{ ms}^{-2}$$

Question 11

A sealed, thermally insulated cylinder contains 1.0 mol of an ideal monatomic gas at 300 K. A frictionless piston rapidly compresses the gas adiabatically to 1/8 of its original volume. If the molar mass of the gas is 4.0 g mol⁻¹, find the root-mean-square speed of the gas molecules (in km s⁻¹).

Ans: 2.7

Sol:

$$T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} = 300(8)^{2/3} = 1200 \text{ K}$$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31)1200}{0.004}} = 2.735 \text{ km/s}$$

Question 12

A solid steel pipe of length $L = 5.0 \text{ m}$ and cross-sectional area $A = 0.01 \text{ m}^2$ is clamped tightly at both ends between two rigid walls at an initial temperature of 15°C. Geothermal steam is suddenly pumped through the pipe, raising its temperature to 215°C. Since the walls are completely rigid, calculate the compressive force exerted on the walls by the expanding pipe in mega newtons (10⁶ N).

(Given: coefficient of thermal expansion $1.2 \times 10^{-5} \text{ K}^{-1}$, Young modulus of steel 200 Gpa)

Ans: 4.8

Sol:

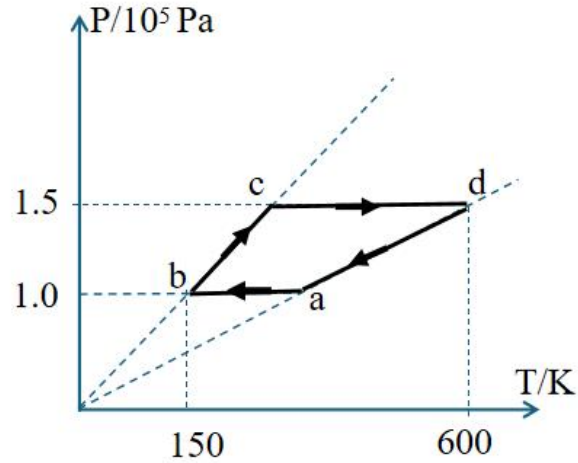
$$\Delta L = \alpha L \Delta T$$

$$F = PA = \frac{\Delta L}{L} EA = \alpha \Delta T EA$$

$$= 1.2 \times 10^{-5} (200) 200 \times 10^9 (0.01) = 4800000 \text{ N}$$

Question 13

The cyclic process, $a \rightarrow b \rightarrow c \rightarrow d \rightarrow a$ shown in the P-T diagram is carried out with 0.8 mol of a diatomic gas. Find the work done per cycle (in kilojoules).



Ans: 0.83

Sol:

$b \rightarrow c$ is a constant volume process with volume $V_b = \frac{nRT_b}{P_b}$

$d \rightarrow a$ is a constant volume process with volume $V_d = \frac{nRT_d}{P_d}$

$$\begin{aligned} \text{So the work done in one cycle } W &= \Delta P \Delta V = (1.5 - 1.0) \times 10^5 (0.8)(8.31) \left(\frac{600}{1.5 \times 10^5} - \frac{150}{10^5} \right) \\ &= 831 \text{ J} \end{aligned}$$

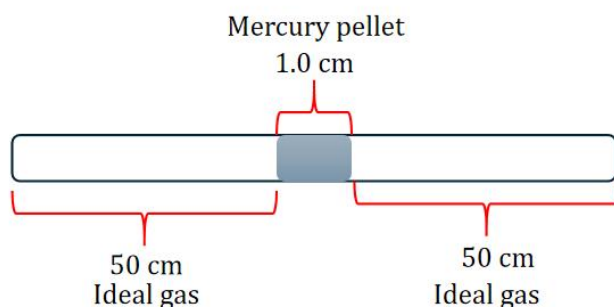
Question 14

A thin, cylindrical glass tube is closed at both ends and placed horizontally on a table. Inside the tube, a mercury pellet of length 1.0 cm divides the tube into two equal compartments, each of length 50 cm. Each compartment is filled with the same amount of an ideal gas with a heat capacity ratio $\gamma = 1.4$.

Initially, both compartments are in thermal equilibrium at the pressure $P_0 = 10^5$ Pa. The glass tube has very small heat capacity and is thermally insulated from its surroundings.

If the mercury pellet is slightly displaced along the tube and released from rest, it will oscillate like a simple harmonic oscillator. Determine the oscillation period in milliseconds.

(Given the density of mercury 13.6 g cm^{-3} .)



Ans: 98

Solution: let the pellet be displaced x to the right initially,

$$P_{Right} = P_0 \left(\frac{V_0}{V_{Right}} \right)^\gamma = P_0 \left(1 - \frac{x}{L_0} \right)^{-\gamma}$$

$$P_{Left} = P_0 \left(\frac{V_0}{V_{Left}} \right)^\gamma = P_0 \left(1 + \frac{x}{L_0} \right)^{-\gamma}$$

$$\begin{aligned} \Delta P &= P_{Right} - P_{Left} = P_0 \left(1 + \frac{x}{L_0} \right)^{-\gamma} - P_0 \left(1 - \frac{x}{L_0} \right)^{-\gamma} \\ &\approx -2P_0\gamma \frac{x}{L_0} \end{aligned}$$

The net force experienced by the pellet is

$$F = -2P_0\gamma \frac{x}{L_0} A$$

$$\text{So, } \omega^2 = \frac{2\gamma P_0 A}{m L_0} = \frac{2\gamma P_0}{\rho L_0 d}$$

$$T = 2\pi \sqrt{\frac{M L_0}{2\gamma P_0 A}} = 2\pi \sqrt{\frac{\rho d L_0}{2\gamma P_0}} = 2\pi \sqrt{\frac{13600(0.01)(0.5)}{2(1.4)(10^5)}} = 0.098 \text{ s}$$

Question 15

Three identical objects are placed at the vertices of an equilateral triangle and connected pairwise by identical springs, each with spring constant k . The springs have unstretched length $l = 9$ cm, so the system is initially in equilibrium in the shape of an equilateral triangle of side 9 cm. Each object is then given an identical positive charge, $+q = 1.8 \mu\text{C}$. The system reaches a new equilibrium in which the area of the triangle is four times its original value.

Assuming the springs are insulating, find the spring constant k in units of N/m.

(The figure is for illustration only and is not drawn to scale.)

Ans: 10

Sol:

The height of the triangle $h\sqrt{l^2 - l^2/4} = \frac{\sqrt{3}}{2}l$

When the area of the triangle increases by 4 times:

$$A_{\text{new}} = 4A_{\text{old}}$$

$$l_{\text{new}} = 2l_{\text{old}} = 18\text{cm}$$

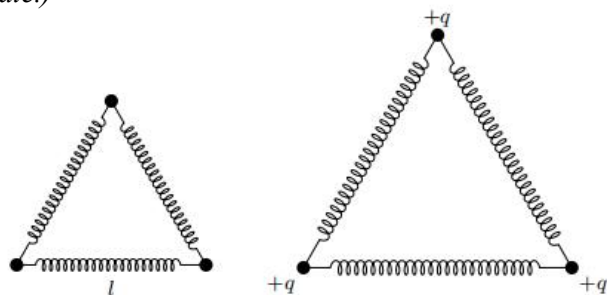
Therefore, the displacement of the spring: $\Delta l = l_{\text{new}} - l_{\text{old}} = 9$ cm.

The large triangle is in equilibrium, if we look at the total force due to the $+q$ on the bottom left vertex (same for the one due to $+q$ on the bottom right vertex) acting on the top $+q$, the repulsive electric force must cancel the restoring spring force:

$$F_e = F_s$$

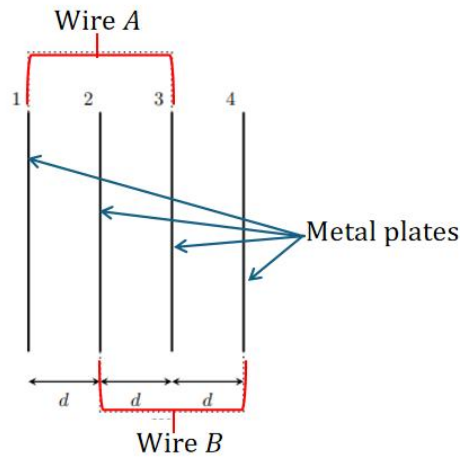
$$\frac{q^2}{4\pi\epsilon_0 l_{\text{new}}^2} = k\Delta l$$

$$k = 8.99 \times 10^9 \times \frac{(1.8 \times 10^{-6})^2}{(18 \times 10^{-2})^2 \times 9 \times 10^{-2}} = 10\text{N/m}$$



Question 16

Consider a capacitor made of four parallel metal plates, each of large area 200 cm^2 , with adjacent plates separated by a small distance $d = 1.0 \text{ mm}$. The first and third plates are connected to the same node A, and the second and fourth plates are connected to node B. Assuming the space between the plates is air (or Vacuum) and edge effects are negligible, determine the equivalent capacitance of the system in nano farads (nF).



Ans: 0.53

Since plates 1 and 3 are connected to node A, plates 2 and 4 are connected to node B, the system is equivalent to 3 identical capacitors parallel connected to nodes A and B

$$A - (1,2) - B$$

$$A - (3,2) - B$$

$$A - (3,4) - B$$

The capacitance of two adjacent parallel plates is $C_0 = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 200 \times 10^{-4}}{0.001} = 1.77 \times 10^{-10} \text{ F}$

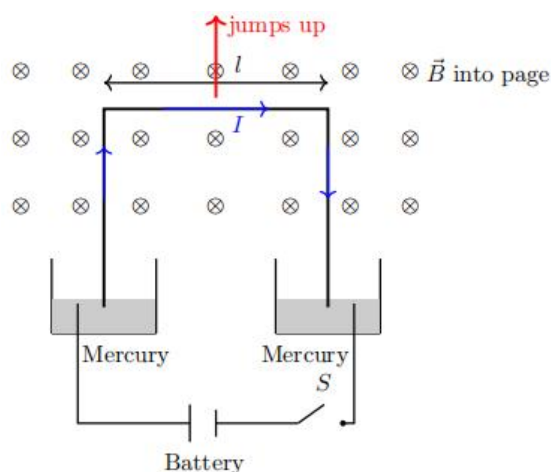
Parallel connection of three identical capacitors gives $C = 3C_0 = 0.53 \times 10^{-9} \text{ F}$

Question 17

An inverted U-shaped conducting wire of total mass m has a horizontal segment of length l . The wire is placed in a uniform magnetic field $\vec{B}$, directed into the page and perpendicular to the plane of the wire. The lower ends of the two vertical segments dip into two mercury tanks, which are connected to an external battery through a switch S . When the switch is closed, current flows through the wire. The wire immediately loses contact with the mercury and “jumps” upward, reaching a maximum height h . Given

$$m = 10 \text{ g}, l = 20 \text{ cm}, h = 3 \text{ m}, B = 0.10 \text{ T},$$

find the total charge q that passes through the wire before it leaves the mercury.



Ans: 3.8

Sol:

The force experienced by this U-shaped wire in external magnetic field:

$$F_B = BIl = \frac{\Delta mv}{\Delta t}$$

$$BIl \Delta t = mv - 0$$

Since all the forces acting in this system is conservative, we have conservation of energy

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

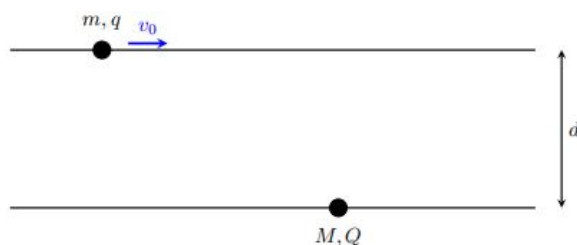
Therefore, we find:

$$BIl \Delta t = m\sqrt{2gh}$$

$$I\Delta t = q = \frac{0.01}{0.1 \times 0.2} \sqrt{2(9.8)(3)} = 3.834 \text{ C}$$

Question 18

Two small beads slide without friction, one on each of two long horizontal parallel fixed rods a distance $d = 0.10 \text{ m}$ apart. The masses of the beads are $m = 0.020 \text{ kg}$ and $M = 0.080 \text{ kg}$, and they carry charges $q = 2.0 \times 10^{-6} \text{ C}$ and $Q = 3.0 \times 10^{-6} \text{ C}$. Initially, the larger mass M is at rest and the other one is far away approaching it at a speed v_0 . What is the minimum value of v_0 (in ms^{-1}) required for the smaller bead to ever get to the right of the larger bead?



Ans: 8.2

For the m to pass the larger bead M , their horizontal coordinates must become equal. At that instant, the distance between the charges is the minimum possible value $r = d$, and the *relative horizontal velocity must be zero*.

Conservation of energy requires

$$\frac{1}{2}mv_0^2 = \frac{1}{2}MV^2 + K\frac{qQ}{d}$$

$$mv_0 = (m + M)V$$

Solving the two equations gives

$$v_0 = \sqrt{\frac{2KqQ(m + M)}{mMd}}$$

$$= \sqrt{\frac{2(9 \times 10^9)(2.0 \times 10^{-6})(3.0 \times 10^{-6})(0.02 + 0.08)}{(0.02)(0.08)(0.1)}} = 8.216 \text{ ms}^{-1}$$

Question 19

Two parallel conducting rails MM' and NN' , separated by a distance $l=0.50\text{ m}$, are fixed horizontally in a uniform magnetic field of magnitude $B=0.20\text{ T}$, directed out of the page. The rails have negligible resistance. Two conducting rods, ab and cd , each of resistance $R=0.10\Omega$, lie across the rails and can slide without losing contact. During motion, rod ab is subject to a frictional force $f_1=0.24\text{ N}$, while rod cd is subject to a frictional force $f_2=0.12\text{ N}$.

A constant horizontal force F is applied to rod ab toward the right. Eventually, the rods move with constant velocities and the system reaches steady uniform motion. Determine the value of F in newtons (N).

Ans : 0.36

For both of the rods to move at constant velocities, the total force experienced by each of the rods should be zero.

For rod ab ,

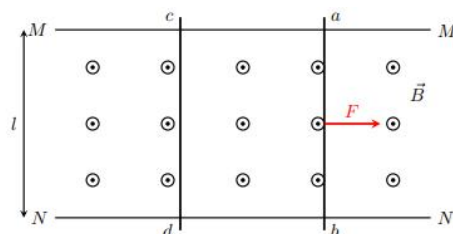
$$Bil - f_2 = 0$$

For rod cd ,

$$F - Bil - f_1 = 0$$

$$F - f_2 - f_1 = 0$$

$$F = 0.36\text{ N}$$

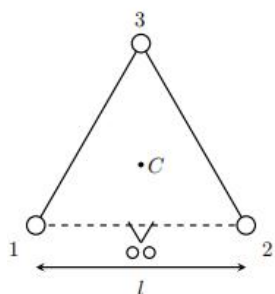


Question 20

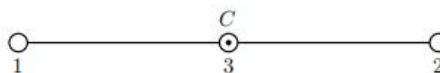
Three identical point charges, each with mass $m = 1.0 \times 10^{-3}\text{ kg}$ and charge $q = 1.0 \times 10^{-6}\text{ C}$ are connected by light insulating, inextensible strings, and form an equilateral triangle with side length $l = 8.3\text{ cm}$. The triangle is initially rest on a smooth horizontal and insulating table as shown in the figure (a).

The string connecting charges 1 and 2 is suddenly cut, while the other two strings remain intact. The three point charges start moving due to electrostatic force. Find the speed of charge 3 when the three charges lie on a straight line as shown in figure (b).

(a) Initial configuration



(b) When charge 3 reaches C



Ans: 8.5

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After the string connecting charges 1 and 2 is cut, the system remains symmetric about the vertical line through charge 3 and the center C . Since all forces are internal, the total linear momentum of the system remains zero.

Moreover, the tensions in the two remaining inextensible strings do no net work on the system, so mechanical energy is conserved.

Initially, all three separations are equal to l . Hence, the initial electrostatic potential energy is

$$U_i = 3K \frac{q^2}{l}$$

At the instant charge 3 reaches C , electrostatic potential energy is

$$U_f = 2K \frac{q^2}{l} + K \frac{q^2}{2l} = \frac{5}{2} K \frac{q^2}{l}$$

Conservation of momentum gives

$$mv_1 + mv_2 - mv_3 = 0$$

Symmetry requires $v_1 = v_2$, so $v_1 = v_2 = v_3/2$

Conservation of energy gives

$$\begin{aligned} \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 + U_f &= U_i \\ 2 \times \frac{1}{2}m(v_3/2)^2 + \frac{1}{2}mv_3^2 + \frac{5}{2}K \frac{q^2}{l} &= 3K \frac{q^2}{l} \\ v_3 &= \sqrt{\frac{2Kq^2}{3ml}} = \sqrt{\frac{2(9 \times 10^9)(1.0 \times 10^{-6})^2}{3(1.0 \times 10^{-3})(0.083)}} = 8.5 \text{ m/s} \end{aligned}$$

Question 21

In a cloud chamber, a uniform magnetic field $\vec{B}$ points into the page. A proton moves in the plane of the page, so that its velocity is always perpendicular to $\vec{B}$. Before any collision, the proton follows a counterclockwise circular trajectory of radius r_1 .

A neutron then enters the chamber, moving along the positive x -axis with velocity u_1 and collides elastically with the proton. After the collision, the neutron moves off with velocity u_2 in a straight line making an angle 45° above the positive x -axis. At the same time, the proton continues to move counterclockwise in the magnetic field along a new circular trajectory of radius r_2 .

Assuming non-relativistic motion, take the masses of neutron and proton are the same ($1.67 \times 10^{-27} \text{ kg}$) determine the ratio $\frac{r_1}{r_2}$.

Ans: 0.62

Conservation of momentum in x direction

$$u_1 - v_1 = u_2 \cos \theta$$

v_1 is the velocity of proton before collision.

Conservation of momentum in y direction

$$0 = u_2 \sin \theta - v_2$$

v_2 is the velocity of proton after collision.

The radii before and after collision

$$r_1 = \frac{mv_1}{qB}, \quad r_2 = \frac{mv_2}{qB}$$

Let the ratio be $a = \frac{r_1}{r_2} = \frac{v_1}{v_2} \rightarrow v_1 = av_2 = au_2 \sin \theta$

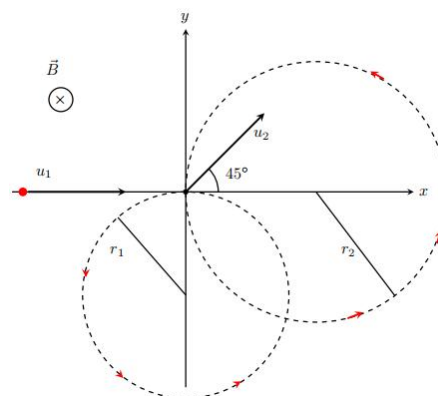
Conservation of kinetic energy gives

$$u_1^2 + v_1^2 = u_2^2 + v_2^2$$

$$u_1^2 + a^2 u_2^2 \sin^2 \theta = u_2^2 + u_2^2 \sin^2 \theta$$

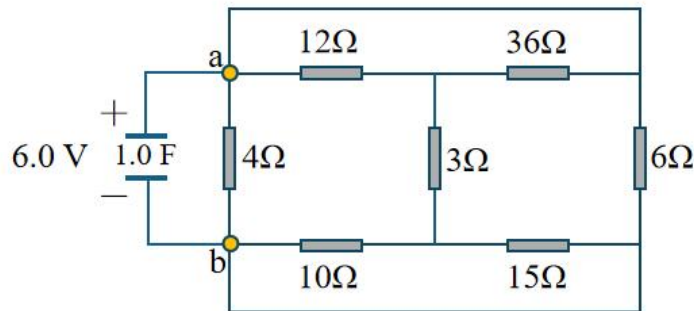
$$a^2 + a - 1 = 0$$

$$a = 0.618$$



Question 22

A capacitor with a capacitance value of 1.0 F is fully charged to 6.0 V , then connected to a network of resistors across points a and b, as shown in the figure. How many seconds does it take for the capacitor to discharge down to 2.0 V ?



Ans: 2.3

First, find the equivalent resistance of the network:

$6\ \Omega$ is parallel with $4\ \Omega$: equivalent = $2.4\ \Omega$

$12\ \Omega$ is parallel with $36\ \Omega$: equivalent = $9\ \Omega$

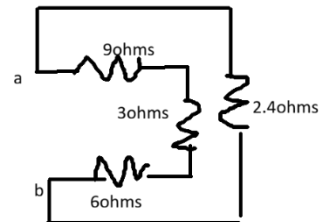
$10\ \Omega$ is parallel with $15\ \Omega$: equivalent = $6\ \Omega$

Reduced circuit:

Effective resistance is thus $2.1\ \Omega$

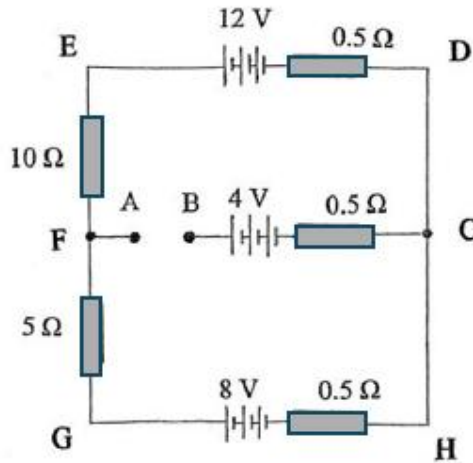
Apply $v(t) = V_0 e^{-(t/RC)}$

$2 = 6 e^{-(t/2.1)}$, hence $t = 2.3$ seconds



Question 23

A circuit is connected as shown in the figure. Calculate the potential difference in volts (voltage) between points A and B.



Ans: 5.4

Zero current flows through the center open circuit branch

Current flowing anticlockwise in the outer loop. Applying kirchhoff voltage law

$$-12 + 10.5i + 5.5i + 8 = 0$$

$$i = 0.25 \text{ A}$$

Assume C is grounded = 0 V (reference)

$$V_A = 12 - 0.25(10.5) = 9.375 \text{ V}$$

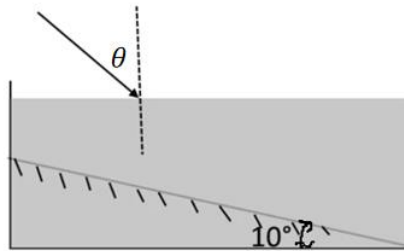
Check: current flowing at bottom = $(9.375 - 8)/5.5 = 0.25 \text{ A} \rightarrow \text{verified}$

$$V_B = 4 + V_C = 4 \text{ V}$$

Hence, $V_{AB} = V_A - V_B = 5.375 \text{ V}$

Question 24

Light enters a water surface at a certain incidence angle θ . The water is contained inside a tank, where, at the bottom of the tank lies a mirror slanting 10° with the base, as shown in the figure. If the light ray exits from the water back into the air, what is the maximum incidence angle θ allowed? Assume the water refractive index is 1.33.



Ans: 40

Use Snell's law

$$n_a \sin \theta = n_w \sin \theta_r$$

The incidence angle in the water to the mirror is given by $\theta_m = \theta_r + 10^\circ$

The incidence angle in the water to the water surface is $\theta_m + 10^\circ = \theta_r + 20^\circ$

At critical angle $\sin \theta_c = \frac{1}{n_w} \rightarrow \theta_c = 48.75^\circ$

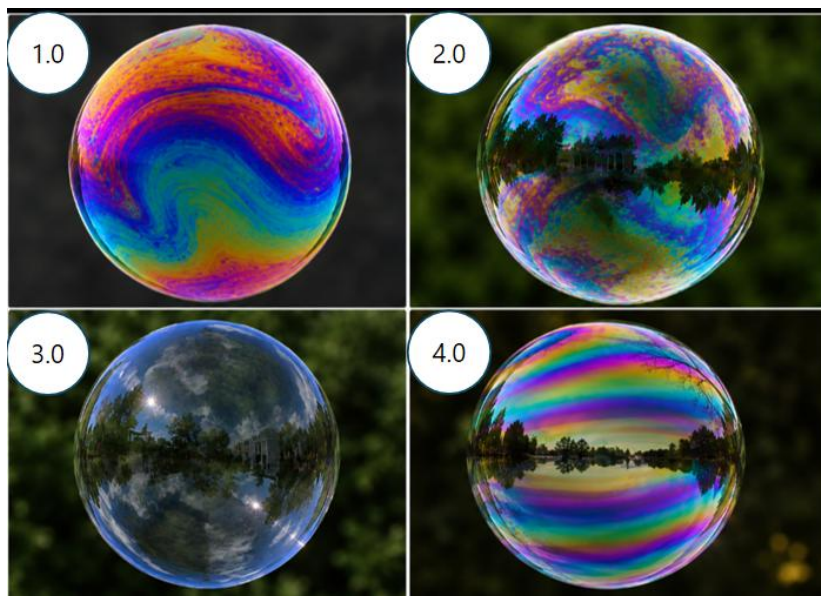
Maximum θ requires $\theta_r + 20^\circ = \theta_c \rightarrow \theta_r = 28.75^\circ$

$$\sin \theta = 1.33 \sin 28.75^\circ$$

$$\theta = 39.77^\circ$$

Question 25

Which one of the four figures most correctly shows the iridescent pattern of a soap bubble illuminated under outdoors sunlight?



Ans: 2.0

1.0: wrong, it is an oil slick pattern

2.0: correct, rainbow pattern that follows the curvature and variable thickness of the bubble surface

3.0: wrong, transparent looks like a solid glass ball

4.0: wrong, although it has rainbow pattern, but it shouldn't be linear as the bubble curvature is spherical and with variable thickness

Question 26

Eric wanted to buy a mirror to see himself from head to toe. He is 1.72 m tall. How tall of a mirror (in meters), at minimum, should he buy?

Ans: 0.86

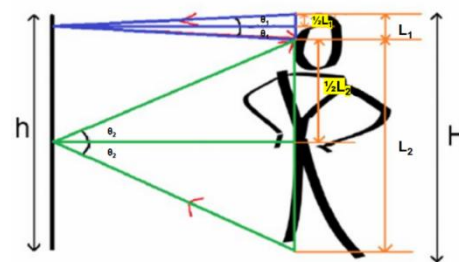
The distance between the two points of reflection on the mirror is:

$$\frac{1}{2}L_1 + \frac{1}{2}L_2$$

Since we know that $L_1 + L_2 = H$, then we can get: $\frac{1}{2}L_1 + \frac{1}{2}L_2 =$

$$\frac{1}{2}H$$

Hence, the minimum tall of the mirror should be $H/2 = 1.72/2 = 0.86$ meters



Question 27

An electron and a tennis ball are both moving at 300 m/s. The ratio of the electron's de Broglie wavelength to the tennis ball's de Broglie wavelength can be expressed as $Z \times 10^{28}$. Take the mass of the tennis ball to be 0.06 kg and the mass of the electron is $9.1 \times 10^{-31} \text{ kg}$. What is the value of Z ?

Ans : 6.6

Solution:

$$\lambda = \frac{h}{mv}$$

$$\frac{\lambda_e}{\lambda_{\text{tennis}}} = \frac{m_{\text{tennis}}}{m_e} = \frac{0.06}{9.1 \times 10^{-31}} = 6.6 \times 10^{28}$$

Question 28

An electron is initially localized in one dimension with a position standard deviation of Δx . The characteristic speed at which the electron's wave packet spreads is estimated by $\Delta p/m$, where m is the electron mass. Applying the Heisenberg uncertainty principle, the value of Δx for which the spreading speed is equal to the speed of light can be estimated as $Z \times 10^{-13} \text{ m}$. What is the value of Z ?

Ans: 1.9

Solution:

From the uncertainty relation $\Delta x \Delta p \leq \frac{h}{4\pi}$

$$\Delta x \approx \frac{h}{4\pi mc} = \frac{6.6 \times 10^{-34}}{4\pi(9.1 \times 10^{-31})(3 \times 10^8)} = 1.92 \times 10^{-13} \text{ m}$$

Question 29

The periodic table has a repeating structure because electrons in atoms can occupy only certain quantized energy states. In a simplified model, suppose that each electron shell with principal quantum number n can accommodate $2n^2$ electrons. How many electrons can the first four electron shells hold in total?

Ans: 60

Solution:

We need to sum contributions for $n = 1, 2, 3, 4$.

$$2(1^2 + 2^2 + 3^2 + 4^2) = 60$$

Question 30

According to de Broglie's hypothesis, a particle with momentum p has an associated wavelength $\lambda = h/p$. Consider an electron moving in a circular orbit of radius r around a nucleus. Assume that the electron's matter wave must form a standing wave around the circumference of the orbit, so that $2\pi r = n\lambda$, where $n = 1, 2, 3, \dots$. Calculate the angular momentum of the electron in units of Planck's constant when $n = 3$.

Ans: 0.48

Sol:

Use de Broglie in the standing wave formula: $2\pi r = nh/p$.

Rearrange to recover the angular momentum $rp = nh/2\pi$.

For $n = 3$, the angular momentum is $L/h = 3/2\pi = 0.4775$